ERROR ANALYSIS OF PRIMARY SIX PUPILS IN WORD PROBLEMS INVOLVING FRACTIONS

ALIYU ALHAJI ZAKARIYYA ADAMU BARWA BEJI
Department of Mathematics, College of Education, Minna, Nigeria
UNOGWU ITODO
Model Science School, Chanchaga, Minna, Nigeria

Abstract

The purpose of this study was to analyze the errors made by primary school pupils in solving mathematical word problems in fraction using Newman's Error Analysis procedure. The study used a qualitative research design and collected data using a diagnostic test and interview. The population of the study was primary six pupils in both public and private schools in Minna metropolis of Niger State, Nigeria. The samples were 105 primary six (6) pupils, 61 males and 44 females. The instruments used in the study are (a) a set of paper and pencil test consisting of 12 theory questions on fractions and (b) structured interview based on Newman's theorem. The diagnostic test was administered to the pupils. Structured interview adopted from Rohmah and Sutiarso was also used to identify at which level students' errors occur. The type of error was based on Newman Error Hierarchy Model that includes reading, comprehension, transformation, process skill, and encoding error. The data were analysed using inferential statistics of Chi-square ($\chi 2$) and Anova. The findings of the study showed that (16.4%) errors are from reading, (19.8%) from comprehension, (18.0%), from transformation while process skills and encoding errors recorded 23.9% and (21.8%) respectively. there is no significant difference between errors committed by the male and female pupils when solving problems involving fraction. It was recommended that teachers should use this procedure as both a pedagogical and remedial tool during their mathematics lesson. This will assist the teachers to be able to identify pupils' errors, assist them in eliminating those errors and encourage them to review their works before submission.

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Introduction

In contemporary educational usage, the curriculum at the school consists of all the experiences that a learner encounters under the direction of the school. Furthermore, the modern concept of curriculum includes the programme of studies, the method of instruction employed in each course, the guidance programme of the school and the extra class-activities programme. The content of the curriculum and the methods employed to present the content have to be determined by the nature of the society, the nature of the learner, and the nature of the subject (Azuka & Kurumeh, 2015). Awofala (2012) identified assessment procedure used in Nigeria schoolsas one of the major handicaps of the content and implementation of the curricula that has been in place during the last 26 years. Nigeria adopts the conventional assessment method where it relied solely on examination results alone to assess the progress and performance of the students. Errors made by the students are often not analyzed on "where" and "why" they made such errors.

Curriculum reform perspectives in mathematics education articulated in many research papers and policy documents aimed at deepening and increasing each learner's mathematical learning and achievement (National Council of Teachers of Mathematics (NCTM), 2014). The perspectives suggest shifts from teacher-centred to

learner-centred approaches. The learner-centred approaches imply that teaching must also be directly responsive to the difficulties experienced by learners on the learning platform, such as the mathematical errors and misconceptions they experience.A key aspect of teaching is the ability of the teacher to make appropriate and reasonable accommodations in order to promote access and attainment for all students. An essential skill for teachers when making these appropriate accommodations is being able to identify where students' misconceptions or errors are preventing them from acquiring new conceptual learning.

Similarly, knowledge of students is one of the three kinds of knowledge that are crucial for teaching mathematics. Others include knowledge of mathematics and knowledge of instructional practices (Hill, Ball, & Schilling, 2008). Knowledge of students and how they learn mathematics includes general knowledge of how various mathematical ideas develop in children over time as well as specific knowledge of how to determine where in a developmental trajectory a child might be. It also includes familiarity with the common difficulties that students have with certain mathematical concepts and procedures, and types of errors they commit while solving mathematical problems.

Yang, Sherman andMurdick (2011) argued that error analysis is an important skill for teachers teaching Mathematics to learners learning in a language other than their home languages, theycalled to the need for a curriculum that supports systematic mastery of mathematical vocabulary, conceptual development and comprehension. Similarly, Luneta and Makonye (2010) hypothesized that poor performance in Mathematics is correlated to learner's errors and misconceptions.

Fractions are among the most complex mathematical concepts that children encounter in their years in primary education. One of the main factors contributing to this complexity is that fractions comprise of multifaceted notion encompassing five inter related sub constructs (part-whole, ratio, operator, quotient and measure).Teaching and learning fractions has traditionally been problematic. In fact, it is well documented that fractions are among the most complex mathematical concepts that children encounter in their years in primary education and hard to be understood by students (Cemalettin &Tuğrul, 2012; Mohyuddin & Khalil, 2016). Similarly, studieshave shown that teachers and pre-service teachershave some difficulties on fraction concept and division in fractionssimilar to those faced by students (Toluk-Uçar, 2009). It was observed that factual knowledge is sometimes being misused in applying procedural knowledge when solving problems. Learners confuse themselves by muddling fraction arithmetic procedures with simple arithmetic knowledge. When calculating$\frac{2}{5}+\frac{1}{3}$, many leaners write down $\frac{3}{8}$ and instead of $\frac{11}{15}$. Similarly, they solved $\frac{1}{2}\div\frac{1}{4}=\frac{1}{2}$ instead of 2. This may be attributed to their immediate access of simple arithmetic knowledge (factual knowledge) instead of applying fraction arithmetic procedures.

Based on the above discussions and evidence from research findings, this paper wishes to contribute to curriculum development by investigating types of errors committed by primary school pupils in Minna metropolis while solving problems involving fraction using Newman's error analysis. The conceptual framework that is used in this study is based on Newman error hierarchical model. The model proposed

by Newman (1977) has been proven to be a reliable model for mathematics teachers to be used to classify and categorize students' errors (Abdullah1, Zainal Abidin& Ali, 2015)

Error Analysis

Error analysis is a type of diagnostic assessment that can help a teacher determine what types of errors a student is making and why. More specifically, it is the process of identifying and reviewing a student's errors to determine whether an error pattern exists that is, whether a student is making the same type of error consistently. If a pattern does exist, the teacher can identify a student's misconceptions or skill deficits and subsequently design and implement instruction to address that student's specific needs. According to Mohyuddin and Khalil (2016) an error takes place when a person chooses the false as the truth. When the actualresult is different from the objective (erroneous result); when the procedure adopted is different from the accepted procedures (erroneous actions) erroneous conceptions might be hindering problem solving and producing irrational results. Errors are of various typesthat include incomplete answer, misused or misread data, technical error, error originating from misconceptions of previously learned material and distorted definition (Schnepper & McCoy, 2013).

Some errors observed include part/part- part/whole error, for example; If in a circle with 10 equal portions, 4 parts are shaded, what fraction of the circle is shaded? The pupils write $\frac{4}{6}$ *instead of* $\frac{4}{10}$. Ordering error, for example when the pupils were asked to order the following fractions: $\frac{2}{3}, \frac{2}{5}, \frac{1}{3}, \frac{2}{7}$, *and write* $\frac{2}{7}, \frac{2}{5}, \frac{2}{3}, \frac{1}{3}$ *instead of* $\frac{2}{3}, \frac{2}{5}, \frac{1}{3}, \frac{2}{7}$.

Benefits of Error Analysis

Error analysis has been shown to be an effective method for identifying patterns of mathematical errors for any student, with or without disabilities, who is struggling in mathematics. Error analysis can be seen as critical analysis of errors in learners' work with a view of looking for possible explanations for these errors. It is a multifaceted activity involving analysis of correct, partially correct and incorrect processes and thinking about possible remediating strategies (Herholdt & Sapire (2014). An error analysis can help a teacher to among others:

Identify which steps the student is able to perform correctly (as opposed to simply marking answers either correct or incorrect, something that might mask what it is that the student is doing right);
Determine what type(s) of errors a student is making;
Determine whether an error is a one-time miscalculation or a persistent issue that indicates an important misunderstanding of a mathematic concept or procedure;
Select an effective instructional approach to address the student's misconceptions and to teach the correct concept, strategy, or procedure (Mohyuddin& Khalil; 2016).

Statement of the Problem

Fractions play an important role in the ever-advancing technological society. Many occupations today rely heavily on the ability to compute accurately, proficiently, and

insightfully with fractions. However, many pupils have difficulty with fractions and computations involving fractions.Due to the nature of the topic (fraction) pupils perform different errors while trying to solve problems involving fraction. These errors include; inability to interpret a word problem, the inability to translate and manipulate signs, inability to read and understand fractions problems. The problems here are: do primary school pupils in Minna metropolis make errors while solving problems involving fraction? And can their errors be classified according to Newman error hierarchical model?

Purpose of the Study

The main purpose of this study is to investigate the type of errors pupils commit while solving problems involving common fractions. Specifically, the study sought to:

1. Find out which of the Newman's procedures (reading, comprehension, translation, process skills, and encoding) pupils are best or worst at when solving problems involving fractions.
2. Find out whether boys and girls are equal in the errors committed while solving fraction problems.

Research questions

The following researcher questions served as a guide to this study:

1. Do pupils commit reading, comprehension, translation, process skills, and encoding errors at the same rate while solving problems involving fraction?
2. Will there be any gender differences in errors committed by pupils when solving fraction word problems?

Research hypotheses

To have the solution to the above stated research questions, the following null hypotheses were formulated.

H_{o1}: There is no significant difference among the types of errors committed by the pupils when solving problems involving fraction.

H_{O2}: There is no significant difference between errors committed by the male and female pupils when solving problems involving fraction.

Research Design

The study is a descriptive study that used both quantitative and qualitative approach. It was design in such a way that a set of paper and pencil testand an interview using the Newman's procedures were administered.

Population

The population of the study consists of 1054 primary six (6) pupils in both public and private schools in Minna metropolis of Niger State, Nigeria.

Sample Size and Sampling Technique

A total of 105 primary six (6) pupils, 61males and 45 females from the selected public and private schools within Minna metropolis were used for the study. The study adopted the use of 10% of the population as suggested by Glenn in Singh and Masuku (2014) to determine the sample.

Multistage sampling technique was adopted as suggested by Teddlie and Yu (2007). There are two major study area, the public and private schools. In the first stage, the stratified sampling was used to group the schools according to public and private schools. In the second stage using systematic sampling, three schools were selected each from each stratum.

Research Instruments

The instruments used in this study were (a)a set of paper and pencil testconsisting of 12 theory questions on fractionsand (b) Structured interview using Newman'serror analysis Procedures. Items in the first instrument covered addition, subtraction, multiplication and division of fractions

The reliability co-efficient of the fraction test was determined using the split-half method. The scores on the odd and even items were subjected to Pearson Product Moment Correlation Coefficient (PPMCC) using Statistical Package for Social Sciences (SPSS). The instrument was found to have reliability coefficient of 0.73.

The Newman's procedures are as follows:

1. Reading level; can the pupils read the question? (simple recognition of words and symbols)
2. Comprehension level; can the pupil understand the meaning of the question? (linguistic and understanding of the problem demands)
3. Transformation level; can the pupil select the appropriate mathematical operations or procedures? (transforming from linguistic understanding to mathematical interpretation)
4. Process skills level; can the pupil perform the mathematical calculation or the procedures accurately? (execution of mathematics processing)
5. Encoding level; can the pupil represent the answer appropriately? (representation of result, from mathematical processing)

The Newman's Error Analysis Interview Prompts consisted of the following:

1. Please read the question to me. If you don't know a word, leave it out.
2. Tell me what the question is asking you to do.
3. Tell me how you are going to find the answer.
4. Show me what to do to get the answer. "Talk aloud" as you do it, so that I can understand how you are thinking.
5. Now write down your answer to the question (Rohmah & Sutiarso, 2018).

Procedures for data collection

The writing of the paper and pencil fraction test was followed by a content analysis of learners written responses in order to extract data from learners' scripts. As suggested by Cheng (2012), sample of the pupils' work for each type of error with at least three items for each error type were collected. The pupils were then asked to verbalize or think aloud as she/he solves the problems without providing any type of cues or prompting. Finally, the scripts were analyze to see patterns among common error types.

Interviews using the Newman's procedures then followed with selection of 20 pupils for further clarity. The data were analyse using chi-square since they are ordinal data as suggested by Rana and Singhal (2015).

Hypothesis Testing

Hypothesis (1) stated that,there is no significant difference in the types of errors committed by the pupils when solving problems involving fraction.

Table 1: Chi-square test (χ^2) of Errors committed by both Private and Public Schools in the Paper and PencilFraction Test

	Value	df	Assump. Sig.
Pearson Chi-Square	0.487	4	0.485
Likelihood Ratio	0.486	4	0.485
Linear-by-linear Association	0.481	1	0.488
Number of valid cases	105		

We can see here that (χ^2) = 0.487, p = .485. This shows that there is no statistically significant difference between private and public schools inerrors committed. That is, both schools equally commit similar types of errors.

Hypothesis H_{O2}: there is no significant difference between errors committed by the male and female pupils when solving problems involving fraction.

Table 2: One-way ANOVA of errors committed by both male and female in both schools

	Sum of squares	df	Mean square	F	Sig.
Gender: B/W groups	0.095	2	0.0475	0.385	
Within Groups	25.467	103	0.247		0.536
Total	25.562	105			
School: B/W groups	2827.361	2	1413.68		
Within Groups	82993.687	103	1		0.064
Total	85821.048	105	805.704		

Table 2 showed one-way Anova of error committed by the pupils by gender. It is used to test the null hypothesis two which states that "there is no significant difference between errors committed by the male and female pupils when solving problems involving fraction. The $f(2, 103) = 0.385, and\ p = 0.536,$ indicated that there was

no significant difference between the errors committed by both boys and girls when solving fraction problems.

Discussion

This study investigated the type of errors pupils commit while solving problems involving common fractions. It was analyzed to find out which of the Newman's procedures (reading, comprehension, translation, process skills, and encoding) primary school pupils are best or worst at when solving problems involving fractions. At the end of the exercise it was observed from the study that pupils' errors were mostly made at the stage of mathematical processing that comprises of transformation, process skills and encoding levels. This finding was in agreement with the findings of Zakaria (2010) which reported that the most frequent errors made by students in using factorization include comprehension error, transformation error and process skill error. A similar result was also obtained from the study of Rohmah and Sutiarso (2018). Their studies showed that 17.39% of the errors occur in comprehension, 34.78% in transformation and 23.91% of the errors occur in process skill errors.

The results from the public school showed that reading is a challenge to the pupils when compared to the pupils from private school. For this group of pupils, it seems that language poses a barrier to them in mathematics tasks presented in English. This however is in disagreement with the finding of Junaedi, Amin, Sugiharti and Chin (2015) whose study concluded that the pupils faced more problem in content-knowledge compared to language difficulties when tackling mathematical tasks in English.

Conclusion

Based on the findings of this study it can be concluded that using Newman procedure to identify the types of error pupils commit while solving word problems involving fraction was found to be useful.Research onanalysis of pupils' error in solving problems involving fraction will be of most benefit to teachers in primary mathematics. The analysis of each pupil's error will enable the teachers to plan their teaching effectivelyand meaningful.

Recommendations

It was recommended that in early grades teachers should give more attention to the basic concepts offractions because they served as fundamental basis for pupils to learn fractions. Pupils need tolearn and experience all definitions of fractions.Training in error management for classroom mathematics teachers should be organized by State Universal Basic Education Boards. Mathematic teachers should always try to analysis their students' scripts, for both wrong and correct answer. This will enable them to correct errors and misconception. They should use this procedure as both pedagogical and remedial tool during their mathematics lesson. That error analysis should be incorporated in all mathematics professional teachers' development programmes. Teaching strategies that involve more of drill and practice should be replaced with reformed approaches that recognise that errors form a valuable source of understanding learners' thinking.

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